SAMA Working Paper

Estimating the Money Demand Function for Saudi Arabia Using Divisia Monetary Aggregate

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Abstract
This paper constructs the broader Divisia monetary aggregate (D2) for the Kingdom of Saudi Arabia over the period from 1999 to 2018. Unlike the traditional money supply aggregate (M2), movements of the Divisia monetary aggregate seems to reflect the domestic economic developments and hence can be very useful when setting macroeconomic policies in the Kingdom. In addition, the paper applies the Keynesian Money Demand Theory to estimate the demand for money using the Divisia monetary aggregate. The findings confirm the stability of the money demand function for Saudi Arabia.

Key words: Divisia monetary aggregate, ARDL, cointegration.
JEL Classification code: C13, C22, E41, E52, F41
1. Introduction

Economists and policymakers have long been focusing on analyzing the behavior of “money” and its interactions with other macroeconomic variables, especially inflation and output. Indeed, deeper understanding of money is warranted to ensure macroeconomic stability.

Central banks and financial authorities publish money supply data in various forms. Traditionally, money supply has been constructed through aggregating monetary assets such as currency in circulation, demand deposits, and other maturity deposits. In many countries, the narrow money supply (M1) is constructed by simply summing up the currency in circulation and demand deposits.\(^1\) Similarly, the broader money supply (M2) is calculated by adding M1 components with longer-term monetary assets (e.g., savings and time deposits).

The traditional aggregation method is based on the underlying assumption of the “perfect substitutability” of all monetary assets within the money supply. While this assumption is suitable for M1 given that both currency in circulation and demand deposits offer zero nominal return, it may

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\(^1\) The name of Simple-Sum money supply is derived from the fact that all components within the monetary aggregate are simply “summed up” together. In notations, the simple-sum monetary aggregate \((S_t)\) is defined as \(S_t = \sum_{i=1}^{N} m_{it}\), where \(m_{it}\) is the monetary component \(i\) at time \(t\).
not be the case for broader monetary aggregates like M2 and beyond. The liquidity utility is higher for currency in circulation and demand deposits compared with savings and time deposits. Using a properly weighted aggregation method can be useful to better understand the money supply.

Broadly speaking, Divisia monetary index is constructed by properly weighting all monetary assets within an aggregate. Unlike the simple-sum aggregation method, the Divisia index is rigorously consistent with the microeconomic and aggregation theories. As highlighted in Alkhareif and Barnett (2012, 2013, and 2015a), Divisia index is among the superlative quantity index numbers. In 2008, amid the global financial crisis, the International Monetary Fund (IMF) has acknowledged the usefulness of Divisia indexes as stated in its Monetary and financial statistics: compilation guide:  

“In constructing broad-money aggregates, it is necessary to evaluate the degree of moneyness of a wide array of financial assets, focusing on the extent to which each type of financial asset provides liquidity and a store of value. Liquidity refers to

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the extent to which financial assets can be sold at, or close to, full market value on short notice.” (MFSM, ¶287)

Many central banks and financial institutions worldwide have utilized in one form or another the techniques of Divisia monetary indexes to help in measuring liquidity. In the case of Saudi Arabia for instance, Alkhareif and Barnett (2013) have used Divisia monetary indexes to predict inflation. They confirmed the usefulness of Divisia indexes when analyzing the business cycle in the Kingdom. Relatedly, Alkhareif and Barnett (2015b) applied Divisia techniques to construct core inflation measures for Saudi Arabia.

Estimating the demand for money is important for many central banks. Indeed, many studies have focused on estimating various forms of money demand functions, including for example Taylor (1993), Hetzel (1984), Goldfeld and Sichel (1990), Mankiw and Summers (1986), Bordo and Choudhri (1982), Juddand and Scadding (1982), and Darrat (1986). Similarly, there are a number of studies that estimated the demand for money using Divisia monetary aggregates, such as Ishida (1984) for Japan, Wesche (1997) for euro countries; Tariq and Matthews (1997) for Pakistan, Khainga

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3 See Banafea (2012) for a comprehensive literature review on demand for money.

Estimating the demand for money in Saudi Arabia has been mostly focused on the traditional monetary aggregates. More recently, Al Rasasi (2016), Hasanov et al. (2017), and Al Rasasi and Banafa (2018) confirmed the stability of money demand function over the long term using simple-sum monetary aggregates. This paper builds on the previous work by utilizing the superlative quantity index, namely Divisia monetary index. To our knowledge, this is the first attempt to estimate the money demand function for Saudi Arabia using the Divisia monetary index.

The rest of the paper is structured as following. Section 2 provides a brief background of the Divisia monetary index along with its application for Saudi Arabia. Section 3 describes the theory for estimating money demand function. Section 4 presents the data sources. Section 5 reports the empirical findings. Finally, section 6 concludes the paper.

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2. Constructing Divisia Money Supply for Saudi Arabia

The Divisia monetary aggregation is constructed on a solid theoretical foundation. In order to construct the Divisia monetary index, one has to calculate the “price” of money or the so-called “user cost”. The user cost is broadly defined as the opportunity cost of holding money rather than the highest-return asset. The following description of the mathematical definition of the user cost and the Divisia monetary index is largely based on the existing literature pioneered by Barnett (1978, 1979a, 1979b, 1980a, 1980b, 1981a, 1981b, 1982, 1987) as well as Alkhareif and Barnett (2012, 2013, and 2015a).

2.1. The User-Cost Formula

Let \( m_{it} \) be the monetary asset \( i \) at time \( t \). In this paper, \( i \) will include all the broader money-supply (M2) components, namely currency in circulation, demand deposits, savings and time deposits. Denote \( R_t \) as the benchmark rate at time \( t \). Here, the benchmark rate would be the interest rate on a given asset that is held solely to accumulate wealth and not for liquidity purposes.\(^5\) Let \( r_{it} \) be rate of return that corresponds to asset \( i \) at time \( t \). Finally, the true cost of living index in time \( t \) is denoted as \( p_t^\ast \). The user-cost formula of asset \( i \) at time \( t \) can therefore be written as following:

\[ U^{\ast}_{it} = \frac{m_{it} \cdot R_t}{R_t + (1 - \frac{1}{R_t}) \cdot m_{it} \cdot (r_{it} - R_t)} \]

\(^5\) In practice, one can choose the maximum interest rate in the universe of all assets in each period. In the case of Saudi Arabia, we use all available information of market interest rates, including on SAMA bills, interbank rates, and other domestic market rates.
\[ \pi_{it} = p_t^* \frac{R_t - r_{it}}{1 + R_t} \]  

(1)

### 2.2. Divisia Monetary Aggregation

Now the user-cost formula is derived, the construction of the Divisia money supply (D2) is straightforward. First, define the expenditure share of asset \( i \) at time \( t \) as following:

\[ s_{it} = \frac{\pi_{it} m_{it}}{\sum_{k=1}^{N} \pi_{kt} m_{kt}} \]  

(2)

Then let \( s_{it}^* = (1/2)( s_{it} + s_{i,t-1} ) \), which represents the average expenditure shares of asset \( i \) corresponding to periods \( t \) and \( t-1 \). The growth rate of the Divisia money supply (D2) can be written as:

\[ \log M_t - \log M_{t-1} = \sum_{i=1}^{N} s_{it}^* ( \log m_{it} - \log m_{i,t-1} ) \]  

(3)

In levels, the Divisia monetary index \( M_t \) is derived as following:

\[ \frac{M_t}{M_{t-1}} = \prod_{i=1}^{N} \left( \frac{m_{it}}{m_{i,t-1}} \right)^{s_{it}^*} \]  

(4)

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6 The continuous time Divisia index is not presented here for simplicity reasons. Similarly, discussion on estimating aggregator functions is beyond this paper to ensure that our results do not entail any estimations that could lead to biased estimates.
Figure 1 depicts the M2 and D2 for Saudi Arabia during the period from January 1999 to December 2018. Movements in the Divisia money supply (D2) seem to reflect the domestic market developments. For instance, the magnitude of the stock market correction that took place in 2006 was properly captured by the Divisia money supply growth rate, which has declined substantially compared with the growth rate of its simple-sum counterpart. Similarly, the broader Divisia monetary aggregate succeeded in capturing the sharp decline in the private sector GDP growth rate that took place in early 2016 on the back of the government arrears. The growth rate of Divisia money supply has rebounded markedly in late 2016 after the remedial measures that aimed at clearing the arrears and improving the liquidity conditions. It is noteworthy that the liquidity conditions have broadly remained balanced in the subsequent years.

Figure 1: Y/Y Growth Rates of Divisia Money Supply (D2) and Simple-Sum Money Supply (M2)

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3. Theoretical Foundation for Estimating Money Demand

Numerous studies have focused on the theoretical foundation for money demand function, with some attempts to identify the key determinants explaining the fluctuations of money demand across the world. These attempts have given rise to a number of important theories pertaining to money demand such as the Quantity theory, Keynesian theory, Inventory Theory, Friedman’s Theory, and Cash-in-Advance approach.\(^8\) Despite the present of various theories, the Keynesian theory continues to dominate the empirical field. Therefore, this paper relies on the Keynesian approach to analyze the behavior

\(^8\) See Banafea 2012 for more details.
of money demand function in Saudi Arabia using the Divisia monetary aggregate.\textsuperscript{9}

The general form of money demand function for Keynesian theory considers two variables, which are the opportunity cost of holding money measured by interest rate, and a scale variable measured by income.\textsuperscript{10} Hence, the money demand could be described as following:

$$\left(\frac{m}{p}\right) = f(y, i) \quad (5)$$

Where $m, p, y, and i$ are the Divisia monetary aggregate, prices level, income, and nominal interest rate respectively; it is essential to note that $\left(\frac{m}{p}\right)$ reflects the real money balance. Since all variables, except for nominal interest rate, are expressed in logarithm form, then we can rewrite equation (5) as follows:

$$m^d_t = \alpha + \beta y_t + \gamma i_t + \epsilon_t \quad (6)$$

where $\epsilon_t$ denotes the error term, while $\alpha, \beta, and \gamma$ are the estimated coefficients. According to the money demand theory, we expect $\beta > 0$

\textsuperscript{9} Based on Keynes' approach, people demand money for transaction, precautionary, or speculative reasons. For the first two reasons, most people tend to demand money for transaction or cautionary reasons leading to higher demand for money implying the positive relationship between income and money demand. For the third reason, people prefer holding financial assets rather than money notably with higher interest rate, and vice versa implying the negative relationship between money demand and interest rate.

\textsuperscript{10} This paper applies the traditional definition of the opportunity cost commonly used in the money demand literature to ensure comparability of results. Other measure of opportunity cost can be investigated in future research.
implying higher income level is associated with higher money demand; on the other hand, we expect \( \gamma < 0 \) meaning that with higher interest rate, people demand less money.

4. Data

Annual data covering the period from 1999 to 2018 are utilized to estimate the Saudi Arabian money demand function. In addition to the broader Divisia monetary aggregate derived in section 2, the non-oil GDP is used as a measure of income, the non-oil GDP deflator is used as a measure of prices, and the US 3-month Libor is used as a measure of opportunity cost of holding money are collected from alternative sources. In particular, the interest rate data are obtained from the St. Louis Federal Reserve Bank, while the remaining data are obtained from the Annual Statistics 2018 published by the Saudi Arabian Monetary Authority (SAMA). All variables are expressed in logarithm form except for the interest rate. To this end, we use the US interest rate as a proxy for the domestic interest rate.

5. Empirical Methodology

5.1. Testing for Unit Root and Cointegration

Numerous cointegration tests have been developed to assess the long-run relationships amid multiple economic variables. The most commonly used
tests are the residual based on Engle and Granger (1987) test, and the maximum likelihood based on Johansen and Juselius (1990) tests. However, these tests have some limitations. For example, these tests require the stationarity of all variables. They also do not perform well for small samples. To overcome these limitations, Pesaran and Shin (1999) and Pesaran et al. (2001) developed the Autoregressive Distributive Lag (ARDL) bounding test for cointegration that has some advantages compared to previous tests. In particular, this test is appropriate for assessing the short and long run relationships for small sample size. Furthermore, this test does not require the stationarity condition implying the robustness of the long-run relationship if it exists. It is also worth noting that the ARDL error correction becomes more efficient when the bound test confirms the existence of a long-run relationship.\(^{11}\)

With this background in mind, it is important to ensure the stationarity of the data before estimating the ARDL model to test for cointegration. More specifically, we apply the Kwiatkowski et al. (1992) test for unit root, in which the null hypothesis is that the series are stationary against the alternative hypothesis of the nonstationarity of the series. The results of the test are

\(^{11}\) Please note that the bound test is based on F statistics (Wald test).
summarized in Table (1). The results confirm that the variables are either integrated of order zero or one, but not two.

| Table 1: Kwiatkowski et al. (1992) Unit Root Test |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| KPSS Test       | Level Data      | First Difference|
|                 | Constant | Trend | Constant | Trend |
| \( m^d \)       | 0.59     | 0.16  | 0.39     | 0.13  |
| \( y \)         | 0.59     | 0.13  | 0.24     | 0.17  |
| \( i \)         | 0.44     | 0.07  | 0.17     | 0.08  |

Note: The KPSS 5% critical values for constant = 0.463, and for trend = 0.146.

Next, we estimate the Autoregressive Distributed Lag (ARDL) Model based on the money demand function given by equation (6), as specified below:

\[
\Delta m_t^d = \alpha_0 + \beta_1 \ln m_{t-1}^d + \beta_2 \ln y_{t-1} + \beta_3 \ln i_{t-1} + \sum_{i=1}^{p} \varphi_i \ln m_{t-1}^d + \\
\sum_{j=1}^{p} \theta_i \ln y_{t-1} + \sum_{i=1}^{p} \mu_i \ln i_{t-1} + \epsilon_t \quad (7)
\]

where \( \Delta \) represents the first difference operator, and \( \epsilon_t \) denotes the error term. The first part of equation (7) with the coefficients \( \beta_1, \beta_2, \) and \( \beta_3 \) represents the long run relationship, while the second part with the coefficients \( \varphi_i, \theta_i, \) and \( \mu_i \) shows the short-term dynamics.

Following Pesaran and Shin (1998), we apply a two-stage procedure. In the first stage, we select the optimal lag length based on Akaike Information Criterion (AIC). In the second stage, we estimate the ARDL Model. Then, we
preform the bounding test for cointegration based on F statistics (Wald test). In particular, we test the joint significance of the estimated parameters on the lagged variables shown in equation (7). Hence, the null hypothesis to be tested is $H_0 = \beta_1 = \beta_2 = \beta_3 = 0$; versus the alternative hypothesis stating that $H_0$ is invalid. We compare the F statistics with the critical values based on Pesaran et al. (2001) to assess whether there is a long-run relationship. If the computed F statistics lies below the lower bound of the critical values, it implies the absence of a cointegration relationship. In contrary, when the F statistics is larger than the upper bound of the critical values, then there is a long-run relationship among the economic variables. When the F statistics lies between the lower and upper bounds, then the long-run relationship will be inconclusive.

The Bounds test results are summarized in Table (2). Our findings confirm the presence of a cointegration relationship between Divisia money demand and its determinants, since the calculated F-statistics is above the upper bound at all significance levels.

<table>
<thead>
<tr>
<th>Test statistics</th>
<th>Value</th>
<th>Significance level</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>12.83</td>
<td>10%</td>
<td>2.63</td>
<td>3.35</td>
</tr>
<tr>
<td>K</td>
<td>2</td>
<td>5%</td>
<td>3.1</td>
<td>3.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.5%</td>
<td>3.55</td>
<td>4.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1%</td>
<td>4.13</td>
<td>5</td>
</tr>
</tbody>
</table>
5.2. Interpretation of the Long-Run Relationship

The estimated long run coefficients of ARDL model are presented in Table 3. It is evident that these parameters are in line with the theoretical foundation. Notably, higher income would be associated with higher money demand, while the rise of interest rate would increase the borrowing costs which in effect will dampen the demand for money.12

Next, we estimate the following Error Correction ARDL model to obtain some insight into how long-run equilibrium is restored between money demand and its determinants, as specified below:

\[
\Delta m_t^d = \alpha_0' + \beta_1' \ln m_{t-1}^d + \beta_2' \ln y_{t-1} + \beta_3' \ln i_{t-1} + \sum_{i=1}^p \varphi_i' \ln m_{t-1}^d + \sum_{j=1}^p \theta_j' \ln y_{t-1} + \sum_{l=1}^p \mu_l' \ln i_{t-1} + \rho ECT_{t-1} + \varepsilon_t \quad (8)
\]

where \( ECT_{t-1} \) is the error-correction term derived from the specified ARDL model, which measures the speed of adjustment in case the demand for money deviates from its long-run equilibrium. The error-correction coefficient is found to be negative and statistically significant. More precisely, the results imply that it takes the demand for money about one year to return to its steady state condition.

<table>
<thead>
<tr>
<th>Table 3: Parameter Estimates</th>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Estimates</td>
</tr>
<tr>
<td>T-statistics</td>
</tr>
</tbody>
</table>

12 Here, the impact of interest rate is statistically insignificant.
5.3. Assessing the Stability of Money Demand Function

We attempt to assess the stability of money demand function to ensure the robustness of our estimates. We perform both CUSUM and CUSUM Squared tests that were developed by Page (1954). Figures 2 and 3 confirm the stability of the money demand function over the long run, since both CUSM and CUSUM-Squared tests do not fall outside the 95 significance intervals. In addition to these tests, Hansen (1992) developed the Lc instability test, which is very useful for assessing the ability of the model in capturing a stable relationship. The null hypothesis of this test is that the parameter stability against the alternative hypothesis of parameter instability. Table 4 shows the result of the Lc test indicating the stability of the parameter estimates at 5 percent significance level. This test also can be viewed as cointegration test in which the null hypothesis of cointegration against the alternative hypothesis of no cointegration.\(^\text{13}\) This in turn confirms the finding of the bounds test, presented in Table 2, for the presence of a long-run relationship between the demand for Divisia money and its determinants.

<table>
<thead>
<tr>
<th>Lc</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.41</td>
<td>0.08</td>
</tr>
</tbody>
</table>

\(^{13}\) See Hansen (1992) for more details.
Figure 2: CUSUM Test

Figure 3: CUSUM Squared Test
6. Conclusion

This paper constructs the broader Divisia money supply (D2) for Saudi Arabia using monthly series from January 1999 to December 2018. Then, we use the modern monetary aggregate (i.e., Divisia Index) to estimate the money demand function for the Saudi Arabian economy following the Keynesian Money Demand Theory. The findings suggest that movements in Divisia monetary aggregate are in line with the domestic economic developments and hence can help in explaining domestic liquidity conditions in Saudi Arabia.

Future research could build upon this paper by applying non-linear econometric techniques and by incorporating additional explanatory variables such as oil prices and government expenditures.
References


